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Investigation of Laguerre expansion basis for modelling of bridge aeroelastic forces

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SUMMARY:
In recent years, great progress has been made in developing models for suspension bridges' aerodynamic behaviour. However, simplified and easy-to-evaluate models are still required for preliminary design and reliability assessment. The present work aims to develop an effective linear model for approximating self-excited forces on bridge decks. The model is designed as a variation on the work of Skyvulstad et al., 2021, using a set of orthonormal Laguerre functions as a basis to express the linear kernel of a Volterra series expansion for the aerodynamic forces. The strengths and weaknesses of the framework are explored on experimental data from the wind tunnel at NTNU in Trondheim, Norway. The model predictions are compared to results using the classical rational functions model. It is concluded that the proposed method is efficient and capable of modelling wind action with similar performances as rational functions in both time and frequency domains. The model is combined with the structural properties of a section model to make a state space model of the aeroelastic system. Good agreement in the predicted instability limits by both methods was found and it is concluded that the proposed framework shows great promise in modelling aerodynamic self-excited forces.

Keywords: Laguerre functions, linear model, aeroelastic forces

1. ORTHONORMAL FUNCTIONS APPROXIMATION

Any kind of continuous real function can be written as an infinite linear combination of orthonormal functions from a complete set $\{g_l(\tau)\}$. For instance, the unit-impulse response of a linear system $h_{nm}(t)$, from the input m to the output n, can be expressed as shown in Eq. (1).

$$h_{nm}(t) = \sum_{l=0}^{\infty} c_l^{nm} g_l(t) \approx \sum_{l=0}^{L} c_l^{nm} g_l(t) \tag{1}$$

Here the $g_l(t)$ is the orthonormal function of order l, which is multiplied by the relative coefficient c_l^{nm}. However, thanks to the properties of complete orthonormal sets, if the system has all poles strictly on the left-half complex plane, the impulse response function can be approximated with a finite summation, as in Eq. (1), with increasing accuracy with the number of terms considered L. The expression of the set of discrete Laguerre orthonormal functions considered in the present work is shown in Eq. (2).

$$g_l(k) = \alpha^{\frac{k-l}{2}} (1-\alpha)^{1/2} \sum_{i=0}^{l} (-1)^i \binom{k}{i} \binom{l}{i} \alpha^{l-i} (1-\alpha)^i \tag{2}$$

The shape of these functions depends on two parameters: the decay factor α, and the filter order l. From the impulse response function, the self-excited force can be expressed relative to the specific input, as in Eq. (3).

$$F_n(j) = \sum_{k=0}^{M} \sum_{l=0}^{L} c_l^{nv} g_l(k) v(j-k) \tag{3}$$

$$\boldsymbol{F}_n = \boldsymbol{S}_v \boldsymbol{c}^{nv} \tag{4}$$

Where $v(j)$ is the input motion convoluted with the sum of filters, evaluated over the memory length M, which determines the regression matrix $\boldsymbol{S}_v$. The expression in Eq. (3) can be expressed in matrix form as in Eq. (4). The model training performances are improved by constructing the $\boldsymbol{S}_v$ matrix with a special recursive relation. Thanks to this, the memory length M is removed as a model parameter, and the effective memory is determined only by the longest stretching filter. The vector of unknown coefficients is then determined by solving the least-squares problem. Once the model is trained, the output can be simulated for any input using Eq. (3). Thanks to the short computational time required for the here exposed training procedure, different values of the model parameters (α and l) can be tested and the modelled force can be compared with the measured one for each case. This will lead to an optimal selection of the parameter values.

2. LANGENUEN EXPERIMENTAL CAMPAIGN

The identified model is been tested on wind tunnel data collected during an experimental campaign in the wind tunnel of the Norwegian University of Science and Technology (NTNU) (Bergerud and Torød, 2021). A special forced motion rig has been used, shown in Fig. 1, capable of forcing the girder model in almost any desired motion in the wind flow. Thanks to the very light computational

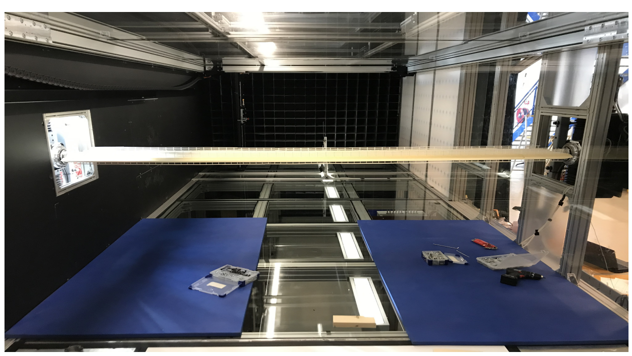

Figure 1. NTNU test vibration rig: experiment preparation.

burden of the presented model, the identification procedure has been calibrated by comparing the predicted time history with the measured data for different values of the model parameters. The time history comparison has been carried out with the *Compmet.m* Matlab toolbox by Kavrakov et al., 2020, which gives the goodness of fit metrics between two time histories. The selected optimal parameter values are $\alpha = 0.6$ and $L = 3$. An example of forced input motion time history and aerodynamic forces can be seen in Fig. 2. In time domain the model performs well and correctly reconstructs the measured forces. The identified Laguerre model has been also compared to the rational functions model, often used for aerodynamic experimental data interpolation. As seen from

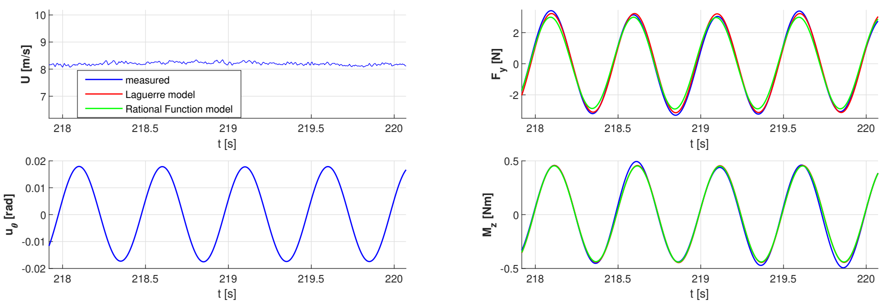

Figure 2. Particular of time histories. Input torsional motion at 2.0Hz. Comparison between measured and modelled forces with rational functions and Laguerre model.

Fig. 2, the difference between the two models in time domain is difficult to appreciate. Therefore a performance evaluation has been carried out also in the frequency domain, comparing the experimental aerodynamic derivatives approximation obtained with the two models. An example of the comparison is reported in Fig. 3. From this latter analysis, small discrepancies between the two models are better highlighted.

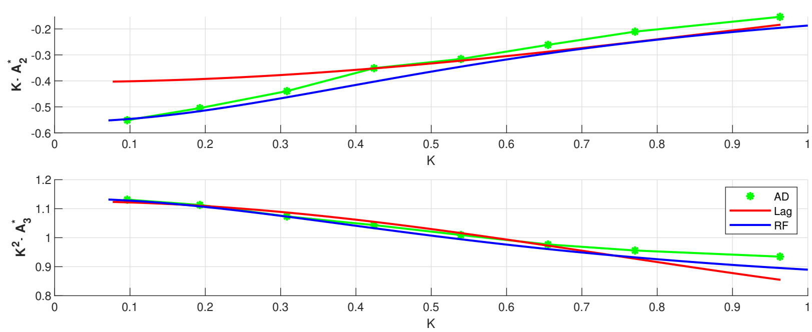

Figure 3. Fitting of aerodynamic derivatives relative to torque because of torsional motion.

3. SIMULATION OF THE DYNAMIC RESPONSE OF A SECTION MODEL

Once the Laguerre model has been trained and proven to perform well for modelling self-excited forces, it was coupled with a simulated two-degrees of freedom section model of a bridge deck. With this aim, a state-space representation for the Laguerre expansion model has been formulated. From the recursive relation used to construct the matrix $\boldsymbol{S}_v$ in Eq. (4) two degrees of freedom discrete state-space Laguerre model has been obtained. The Laguerre state-space model is then inserted in the equations of motion of the deck's dynamical system. The continuous time equations of motion of the two degrees of freedom deck section are rewritten in discrete time since the force prediction model has been developed in discrete time. Therefore, the complete system state space model is obtained as in Eq. 5.

$$\boldsymbol{X}(k+1) = \boldsymbol{AX}(k) + \boldsymbol{BC}\hat{\boldsymbol{S}}(k) \tag{5a}$$

$$\hat{\boldsymbol{S}}(k+1) = \hat{\boldsymbol{A}}_L \hat{\boldsymbol{S}}(k) + \begin{bmatrix} \hat{\boldsymbol{B}}_L & \boldsymbol{0} \end{bmatrix} \boldsymbol{X}(k+1) \tag{5b}$$

Here the regression matrix $\hat{\boldsymbol{S}}$ is constructed with convolution between the input motions in the state vector $\boldsymbol{X} = [y(t), \theta(t), \dot{y}(t), \dot{\theta}(t)]^T$ and the Laguerre filters (Eq. (5b)). This matrix is then multiplied by the set of filters' coefficients in $\boldsymbol{C}$ to obtain the self-excited forces vector $\boldsymbol{F}$. Which is inserted in the system's equations of motion (Eq. (5a)).
Using the complete state-space model, time simulations have been carried out for different wind speeds. The wind speed was increased until the system showed unstable motion. From the time histories in Fig. 4, it can be seen that when the wind speed exceeds $V = 37.6m/s$ the motion starts to diverge because of flutter instability. The predicted dynamic behaviour has been compared with the one found with a state-space model constructed with rational function identification method for self-excited forces. The two models agree on the value of critical flutter speed, the eigenvalues' trend, and the trend of damping and frequency relative to the first two eigenvectors.

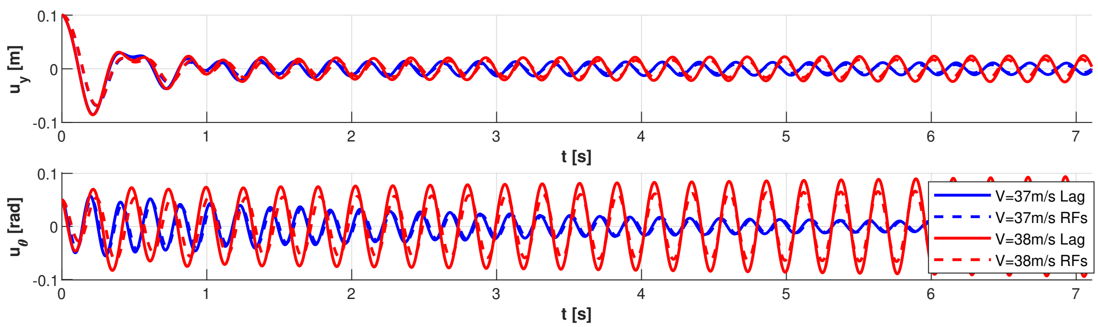

Figure 4. The coupled dynamic model has been simulated for different speeds until instability was observed. Flutter speed $V = 37.6m/s$.

4. CONCLUSIONS

The present work has developed an effective parametric linear model for aeroelastic forces prediction. This has been done by approximating the system transfer functions with a linear combination of orthonormal Laguerre filters. Thanks to the low computational burden an experimental optimisation of the two model parameters has been carried out. The effectiveness of the model has been verified on experimental wind tunnel data from NTNU. The model performances have been compared with the rational functions model, and the two models showed comparable training computational burden and performed similarly in modelling the aerodynamic behaviour of a two degrees of freedom deck section. The presented Laguerre expansion model gives interesting results and has room for further improvement. In the model definition could be added two feed-through terms relative to speed and displacement to improve the impulse response approximation. Furthermore, the training procedure could be improved by training a different set of α and L parameters for each transfer function in the system. Finally, it would be interesting to investigate the eventual differences in performances using white noise as training data instead of a single harmonic motion.

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